Int. J. Heat Mass Transfer. Vol. 36, No. 7, pp. 1969–1980, 1993
Printed in Great Britain

0017-9310/93 $6.00+0.00
Pergamon Press Ltd

On the stability of stratified flow and its transition to other flow regimes

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(*Received November* 1990)

Abstract—Stratified two-phase flow in circular pipes has been treated extensively in the literature. The proposed presentation is based on the author's own paper (L. N. Persen, Stratified two-phase flow in circular pipes, *Int. J. Heat Mass Transfer* **27**, 1227–1234 (1984)) which has the advantage of explicitly expressing the influence of the five parameters that govern the physical process. The investigation will show how the normal depth is determined, that it may be single-valued or double-valued, whereby the possibility of hydraulic jumps is present. It will be shown under what conditions the normal depth is stable or unstable. In the first case the stability may be conditional, i.e. it may be stable against small disturbances but not against large ones, and accurate upper limits for a permissible disturbance will be given. (Large disturbances may be caused by the geometry of the pipeline.) In the second case the transition to another flow regime will be discussed and it will be shown that instability does not necessarily lead to slug flow. Several problems connected with stratified flow in sloping pipelines will be considered. In particular it will be shown how pipeline geometry will affect the flow. Downward sloping pipelines in the direction of the flow will be discussed in detail. Upward sloping pipelines will also be discussed but in that case the stability of the flow may be overruled by geometrically induced slug flow.

INTRODUCTION

STRATIFIED two-phase flow in circular pipes has been treated extensively in the literature. Instability of this type of flow has been the object of scrutiny by many authors [1–5]. Most of these contributions have centred on the Kelvin–Helmholtz type of instability, an approach that seems to resist incorporation of the many parameters that influence the flow.

The proposed approach is based on an entirely different concept which leaves room for variation of the slope angle of the pipe, of the overall pressure under which the flow occurs and of the volumetric flows of gas and liquid, and has explicit parameters for the force transmission at the interface. It is based on the author's own paper [6], which has the advantage of explicitly expressing these influences through the five parameters that govern the physical process. The investigation shows how the normal depth is determined, that it may be single-valued or double-valued, whereby the possibility of hydraulic jumps is present. It will be shown under what conditions the normal depth is stable or unstable. In the first case the stability may be conditional, i.e. it may be stable against small disturbances but not against large ones, and accurate upper limits for a permissible disturbance will be given. In the second case the transition to another flow regime will be discussed and it will show that instability does not necessarily lead to slug flow.

The question of the flow's stability is not entirely a question of the instability of the flow as such, but extraneous influences such as geometrically induced disturbances may enter the problem. In particular it will be shown how pipeline geometry may affect the flow for pipelines which have an upward slope.

It will also be shown how the problem of finding the normal depth and determining its stability will be modified if the pipeline is so long that the pressure reduction due to the pressure gradient has to be accounted for.

THE THEORY

As mentioned, the present approach differs from those previously found in the literature. It is based on the author's publication [6] and may be characterized as a modification of the usual hydraulic approach to open channel flow. The reader is referred to the original paper for the actual deductions, but expressed in the same notations as originally used, the end result as far as the liquid depth is concerned is the differential equation for the liquid depth as a function of the downstream distance:

$$\frac{\mathrm{d}h^+}{\mathrm{d}x^+} = \frac{\sin\alpha - \dfrac{fh_0^+}{8}\left[\dfrac{1}{(F_L^+)^2 R_L^+} - \dfrac{P_2 P_1^2}{(F_G^+)^2 R_G^+} \pm \dfrac{P_4}{s_i^+}\left(\dfrac{1}{F_L^+} - \dfrac{P_1}{F_G^+}\right)^2 (1+P_2)\right]}{1 - h_0^+\left[1 + P_1^2 P_2 \left(\dfrac{F_L^+}{F_G^+}\right)^3\right]\dfrac{\sin\phi}{(F_L^+)^3}}. \tag{1}$$

The notation is given in the appendix and the parameters are defined as follows:

$P_1 = Q_G/Q_L$	the volumetric flow ratio; used as independent non-dimensional variable (abscissa) in all diagrams
$h^+ = h/R$	non-dimensional liquid depth at the centre plane; used as ordinate in the diagrams
$P_2 = \rho_G/\rho_L$	density ratio for gas/liquid. Through this parameter the influence of the overall pressure, under which the flow occurs, is expressed
$P_4 = f/f_i$	ratio between the friction factor expressing the force transmission at the interface and the overall friction factor of the flow
α	slope angle of the pipe (assumed constant along the pipe) $\alpha < 0$ up-flow, $\alpha > 0$ down-flow
f	overall friction factor for the flow (see later remarks)
$h_0^+ = (2Q_L^2/gR^5)$	this parameter expresses the influence of the pipe size (through R) and of the volumetric liquid flow (through Q_L). It is related to the 'head loss' in pipeline design.

The five last quantities are the ones that will be varied in order to investigate their impact on the stability of the flow.

THE STABILITY CRITERION

Equation (1) gives the slope of the gas/liquid interface as a function of its position (h^+). It means that given the liquid depth at one point, the differential equation tells the slope of the interface as one proceeds downstream of this point. Rewriting equation (1) as

$$\frac{dh^+}{dx^+} = \frac{F(h^+, f, h_0^+, P_1, P_2, P_4, \alpha)}{G(h^+, h_0^+, P_1, P_2)}, \qquad (2)$$

one discovers immediately that the liquid depth (h^+) may have a value for which

$$F(h^+, f, h_0^+, P_1, P_2, P_4, \alpha) = 0. \qquad (3)$$

This value is called the normal depth (h_n^+) and represents the equilibrium position of the interface. Once given this value, it is retained downstream and one has a uniform stratified flow. Whether this special equilibrium position is stable or unstable is determined by the differential equation (2) when considering the effect that a deflection away from this position will have:

1. if for any reason the interface is moved above the normal depth and $dh^+/dx^+ < 0$ the interface will return to the normal depth and it is stable;
2. if for any reason the interface is moved below the normal depth and $dh^+/dx^+ > 0$ the interface will return to the normal depth and it is stable;
3. if the opposite is true the interface will move away from the normal depth (in both cases) and the normal depth is unstable;
4. cases may occur where the normal depth is stable against small deflections but unstable against larger ones. This will be discussed subsequently.

The values of h^+ for which the instability sets in are found from a study of the differential equation (2). Figure 1 shows a possible situation where the field $0 < h^+ < 2$ is divided into three regions where the borderline between them is given by either $F = 0$ or

$$G(h^+, h_0^+, P_1, P_2) = 0. \qquad (4)$$

Equation (4) indicates a singular behaviour of the interface. In such a case $dh^+/dx^+ \to \infty$, which usually indicates a hydraulic jump. It may also indicate the onset of slug flow. The stability question is thus reduced to a scrutiny of the behaviour of the G-function in (4) as the liquid depth is moved to varying positions (given by h^+). As in the classical approach this represents giving the solution a perturbation whereupon the stability question is judged from the liquid depth's further development.

It is interesting to note how the different parameters governing the flow will influence the stability. This is brought out by studying equation (4). It is noticed that the solution to this equation is only influenced by the parameters P_1, P_2 and h_0^+. This means that the unstable domains are uninfluenced by the friction factors (the parameters P_4 and f) and the slope of the pipe.

THE NORMAL DEPTH (HORIZONTAL PIPE)

As already mentioned the normal depth is determined from equation (3). However, in the simplifying case of a horizontal pipe, $\sin \alpha = 0$ which means that the normal depth is in this case uninfluenced by f, h_0^+ (which occur in only one combination) and depends solely on P_1, P_2, and P_4. One will thus for this specific case have a simpler situation, three of the parameters having been ruled out.

THE VARIATION OF THE PARAMETERS

The many parameters governing the problem prohibit an easy description of the results in one diagram only. The results are therefore presented in a diagram where the liquid depth (h^+) is the ordinate and the parameter P_1 is the abscissa. Keeping the other parameters constant one may in this diagram plot the normal depth (h_n^+) as a function of P_1. One may further plot the curves through points where equation (4) is satisfied and in this way establish the borderline for the instability region. This will depend on the parameter h_0^+ (representing the volumetric liquid flow), and by varying this parameter one obtains a family of curves giving the instability regions for the various values of h_0^+.

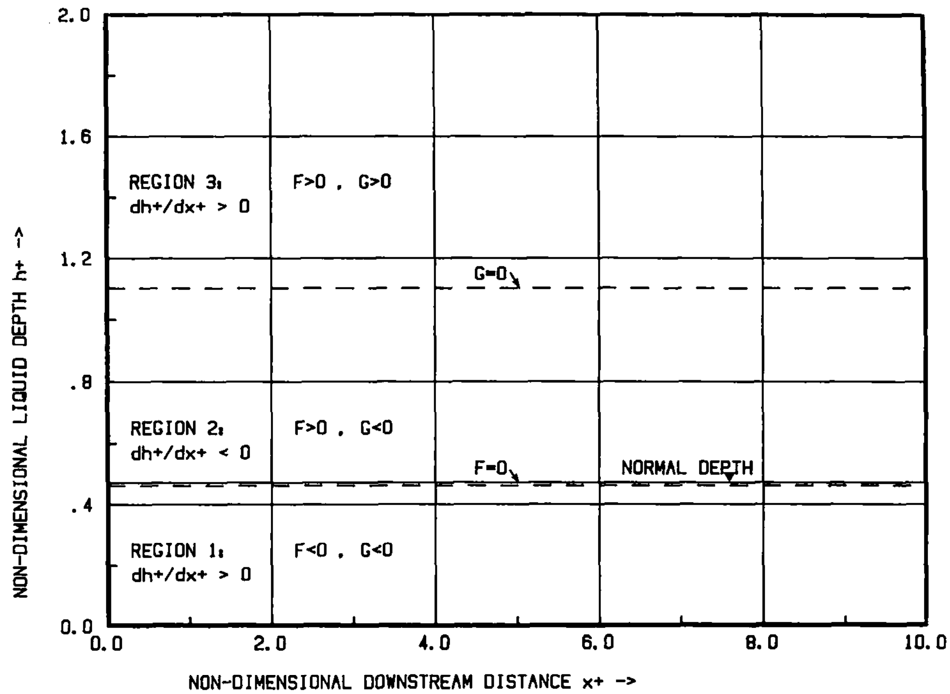

FIG. 1.

The parameters to be kept constant are thus:

The density ratio $P_2 = 0.00121$ (this means that the system is equivalent to an air/water system at standard conditions: 1 atm., 20°C).

The slope angle $\alpha = 0$ (this means the influence of the slope of the pipe is given by comparison to the horizontal pipe).

The friction factor ratio $P_4 = 1$ (this is a value which is thought of as being rather acceptable. Its influence on the results is shown by a variation in this value).

The friction factor $f = 0.025$ (this means that a value corresponding to a rough pipe with a high roughness has been chosen. Since this parameter always appears multiplied by h_0^+, and the latter is varied, there is no need for a variation of this parameter).

THE HORIZONTAL PIPE

Figure 2 shows the normal depth h_n^+ as a function of the parameter P_1. It also shows the regions of instability for different values of the parameter h_0^+. Since an increase in P_1 with a constant value for h_0^+ means an increase in the volumetric gas flow at constant volumetric liquid flow, the diagram reveals that one may always find a stable situation (irrespective of the magnitude of an eventual disturbance) at high enough values of P_1. This means, however, that one gradually approaches the annular-mist-flow-regime, and that no special physical criterion determines the borderline between the two regimes in a map.

For lower values of P_1 it is observed that a range of values exists for which the normal depth is stable for small disturbances but unstable for larger ones. The magnitude of such instability causing disturbances can for the case in question (value of h_0^+) be read out of the diagram.

Finally, a region exists for low values of P_1 where the normal depth is unstable for any disturbance. This region can also be read out of the diagram.

This situation will now be the reference for all further variations of the different parameters.

The first variation to be investigated is the influence of the overall pressure under which the flow occurs. This is especially important in the case of subsea production lines in petroleum engineering where pressures of 100 atm. or more may be found. P_2 is the parameter which is influenced here by the pressure through the effect that it has on the gas density. Thus Fig. 3 is plotted with the same values of all parameters (in Fig. 2) except P_2 which is raised by a factor of 100 (from 0.00121 to 0.121).

It is noticed (by comparison with Fig. 2) that the effect of high pressure is to repress the unstable regions, i.e. to generally stabilize the flow pattern. However, the actual value of the normal depth is also affected, which can easily be seen by comparison.

As stated above the following parameters have been kept constant in making this comparison:

P_4, which expresses force transmission across the interface. No exact value is known for this parameter, but it has been made plausible that a value of 1 may be used. To bring out its importance, Fig. 4 shows the influence a factor of 4 has on the normal depth. As

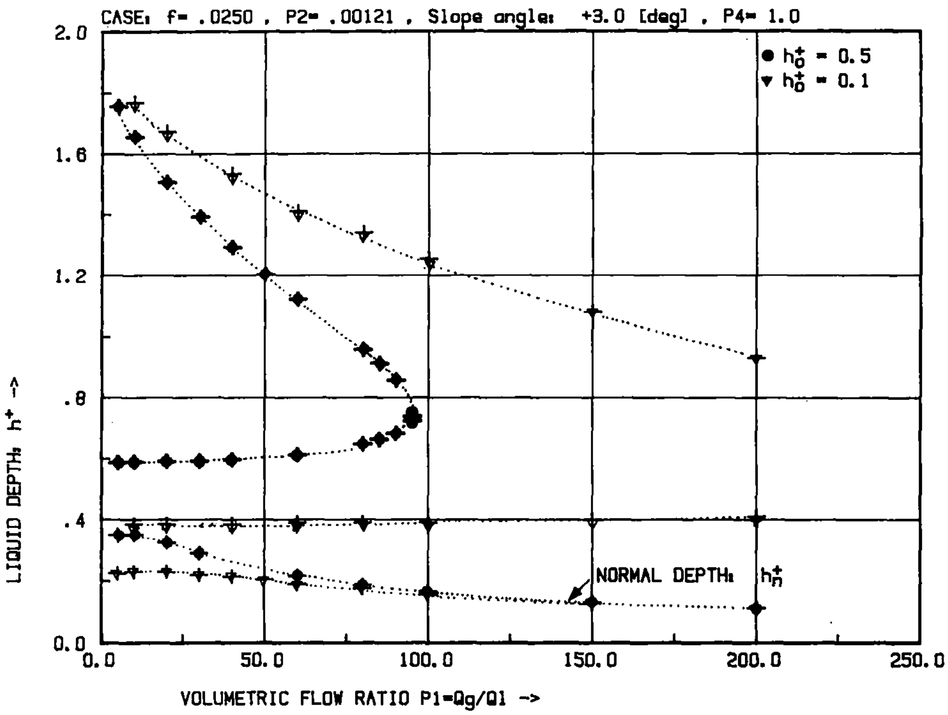

FIG. 5.

ever, mean that backflow is adequately accounted for.)

Finally, it is observed that the normal depth is no longer independent of the parameter h_0^+, a fact that is exhibited by different curves for different values of h_0^+ for both the normal depth and its corresponding instability region.

The situation is drastically changed as the influence of a high overall pressure is examined by raising the P_2-value by a factor of 100. As already observed the

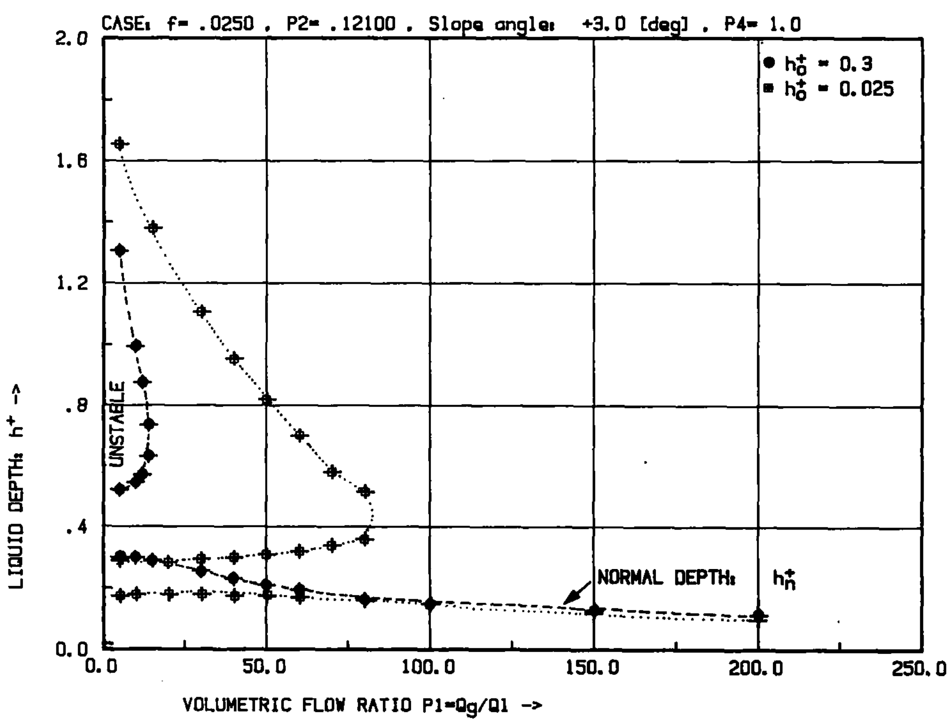

FIG. 6.

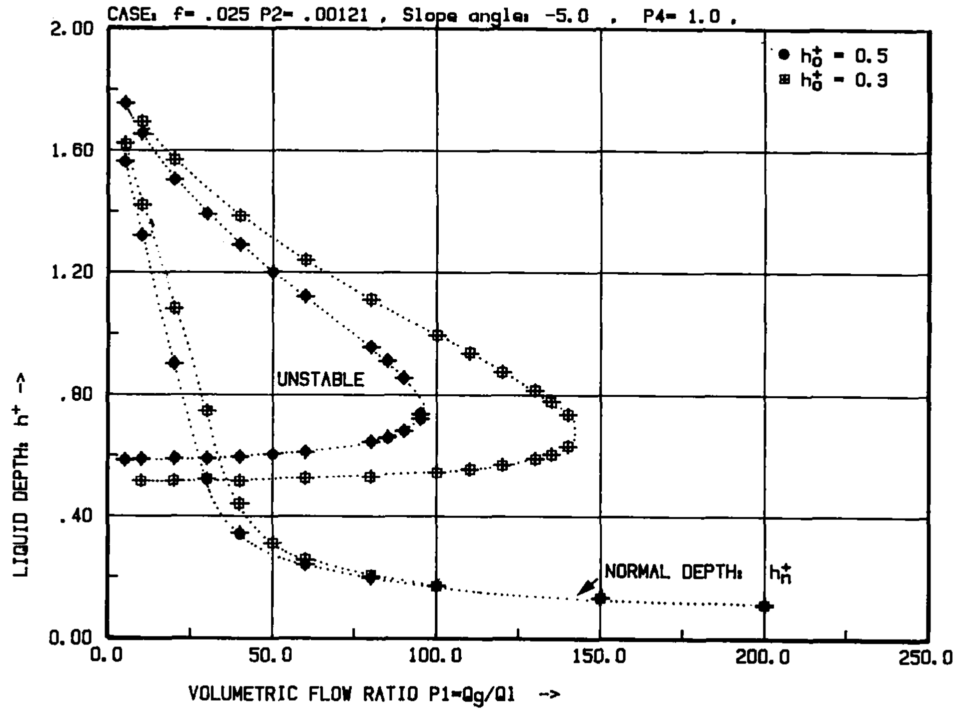

FIG. 7.

effect is to stabilize the flow but in the case exhibited in Fig. 8 the effect is to make the normal depth stable against small deflections in all events. In contrast to all previous cases the situation is now such that only large deflections downwards will cause instability to occur. Further discussion of this situation is deferred to the examination of what happens once instability has set in.

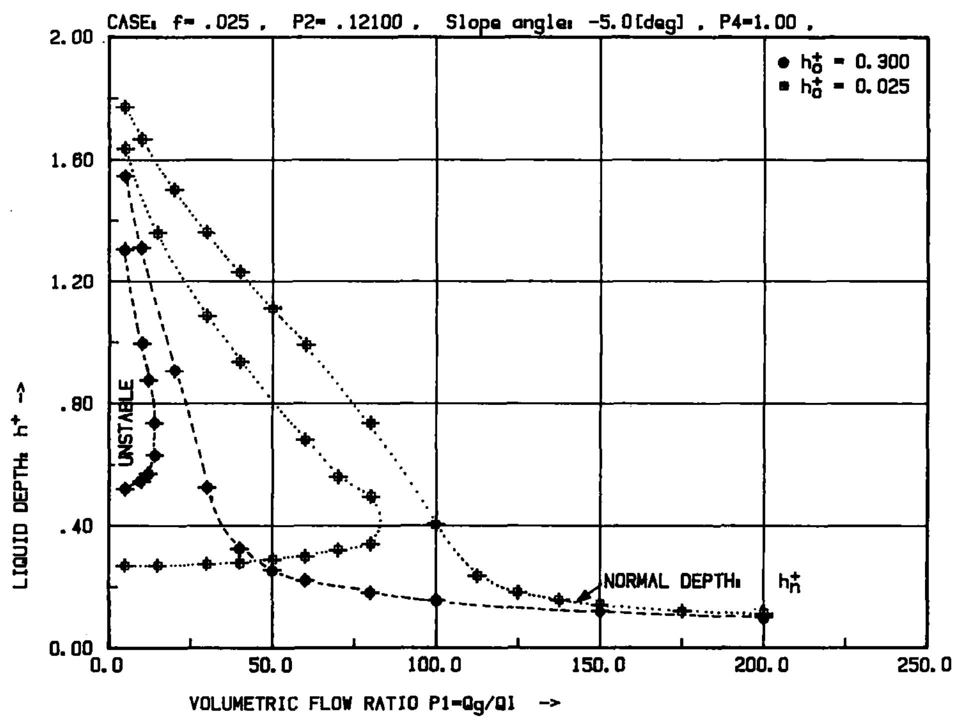

FIG. 8.

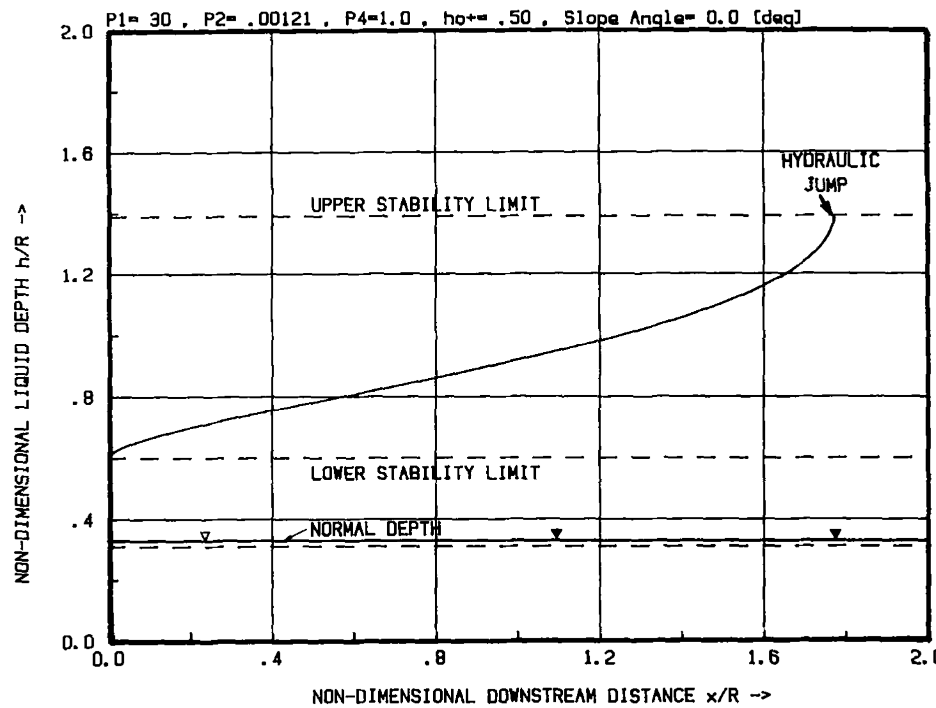

FIG. 9.

THE POST-INSTABILITY LIQUID DEPTH

The question of what happens once the necessary disturbance (deflection of the interface) has occurred can now be discussed. Figure 9 illustrates a situation where reference is made to the point in Fig. 2 where $P_l = 30$, $h_0^+ = 0.5$. It exhibits the way in which the location of the interface develops downstream of a point ($x^+ = 0$) where the interface has been brought into the instability region. It shows that over a distance less than the diameter of the tube the interface goes into a hydraulic jump. What happens thereafter depends on the position of the upper limit of the instability region. If this is close to the top of the pipe a slug will be created. Otherwise the hydraulic jump will move downstream, sometimes dying out as it moves but sometimes also being overtaken by another, creating a slug. The essence of this is that there are several scenarios and that no certain statement can be made, which is in close agreement with the sometimes frustrating experimental situation when regularity of the unstable flow seems to escape the experimenter.

The feature of instability exhibited in Fig. 9 is common to all cases where the region of instability lies above the normal depth. Figure 8 shows a situation where the opposite is the case. To illustrate the post-instability behaviour of the interface in this case attention is drawn to Fig. 10. The case $P_l = 60$, $h_0^+ = 0.025$ is used as an example, which shows how the liquid depth quickly decreases, ending in a singularity in less than a diameter distance downstream. The physical significance of this is a collapse of the stratified flow, a continuation of the downstream flow being outside the grasp of the present approach.

A final word ought at this point to be said about the large deflections needed for the flow to become unstable in certain situations. Such large deflections may occur in pipeline systems consisting of segments with different slopes. Persen [7] has shown how geometrically induced waves with large amplitudes may travel upstream and finally destabilize an otherwise stable stratified flow. Figure 11 is a reproduction of such a case from ref. [7] and it shows how a slug is being formed within a time period of 0.08 s in a 2″ pipe.

COMPARISON WITH PREVIOUS RESULTS

The present results may now be easily compared with the results of previous authors. They all give a stability criterion whereby either the gas velocity or the difference between the gas and the liquid velocity is given an upper limit beyond which instability occurs. Their results will here be restated in the notations used in the present text. The following three criteria may be formulated as one:

Kelvin–Helmholtz criterion [8]: $c = 1.00$

Mishima–Ishii criterion [3]: $c = 0.487$

Wallis–Dobson criterion [5]: $c = 0.50$

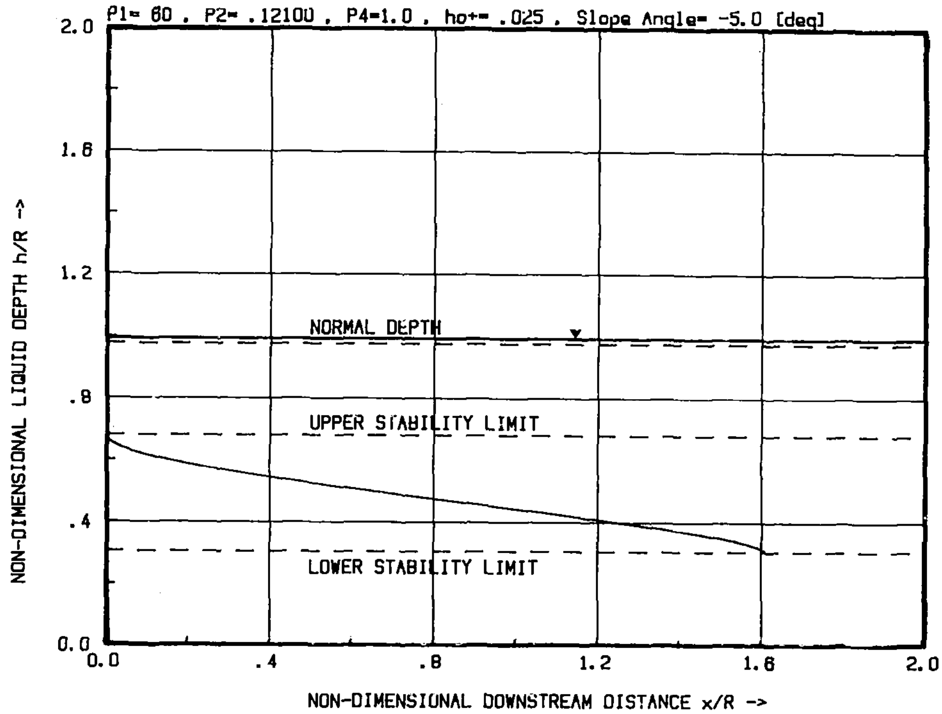
P1= 60 , P2= .12100 , P4=1.0 , ho+= .025 , Slope Angle= -5.0 [deg]
NORMAL DEPTH
UPPER STABILITY LIMIT
LOWER STABILITY LIMIT
NON-DIMENSIONAL LIQUID DEPTH h/R ->
NON-DIMENSIONAL DOWNSTREAM DISTANCE x/R ->
2.0
1.6
1.2
.8
.4
0.0
0.0
.4
.8
1.2
1.6
2.0

FIG. 10.

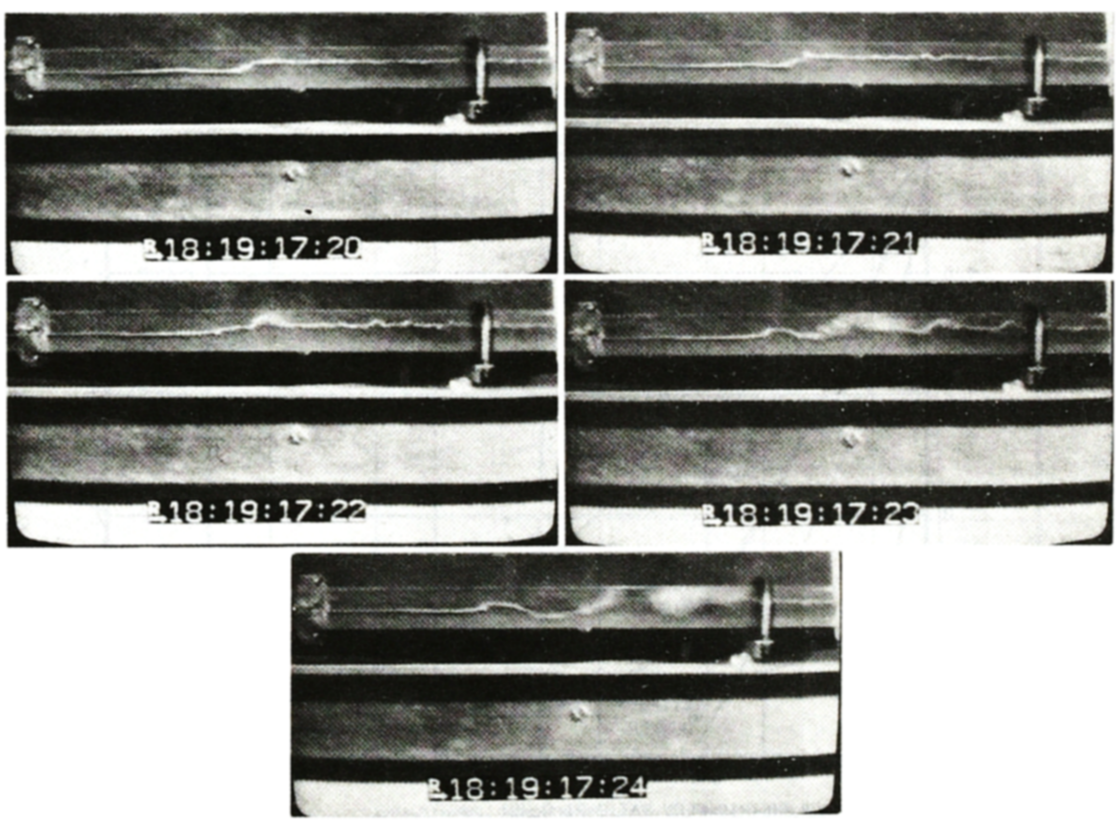
18:19:17:20
18:19:17:21
18:19:17:22
18:19:17:24

FIG. 11.

Criterion:

$$v_G - v_L \geqslant c\sqrt{(\rho_L - \rho_G) g h_G / \rho_G} \tag{5}$$

Reformulation:

$$P_1 \geqslant \left(\frac{1}{F_L^+} + c\sqrt{(1/P_2 - 1)(2 - h^+)/h_0^+}\right) F_G^+. \tag{6}$$

Taitel–Dukler [4]:

Criterion:

$$v_G \geqslant (1 - h_L/2R)\sqrt{(\rho_L - \rho_G) g h_G / \rho_G} \tag{7}$$

Reformulation:

$$P_1 \geqslant (1 - h^+/2)\sqrt{(1/P_2 - 1)(2 - h^+)/h_0^+}. \tag{8}$$

When comparing these results with the present ones, it should be observed that only cases with a slope angle equal to zero can be considered since the previous results are only valid for horizontal pipelines.

For the horizontal pipeline only the low and the high pressure cases are of interest. Figures 12 and 13 show these cases. The previous results are only compared to the present for one chosen value of h_0^+.

The results are seen to incorporate exactly the quantities previously identified as the only ones that affect the instability regions. The results can thus be compared in the same diagram.

First it is noticed that the curves drawn to exhibit the previous results are to be interpreted such that instability occurs when the position of the interface (h^+) for a given value of P_1 is above and to the right of the respective curve. It means that the curve divides the diagram in two: unstable above the curve (and to the right), stable below the curve (and to the left). This means that the previous results all fail to recognize the lower branch of the instability criterion encompassing the instability region. This leads to the flaw in the previous results which all indicate instability for increasing values of the volumetric gas flow (increasing P_1), whereas the reality is that a smooth transition to annular flow takes place. The present approach exhibits this behaviour.

A closer scrutiny of the diagrams reveals that the Kelvin–Helmholtz curve seems to follow the upper boundary of the instability region for both the high and the low pressure case. One should not, however, draw the conclusion that the Kelvin–Helmholtz curve agrees with the present approach. The contrary is the case since the two approaches predict opposite behaviour.

For all previous cases one finds that under some circumstances their prediction of instability agrees with that of the present approach. Their failure to recognize the lower branch cannot, however, be ignored.

Finally, the stabilizing effect of the higher pressure already discussed in connection with Fig. 2 is evident in Fig. 12 also for the results of the previous authors. However, because of the discrepancy mentioned above this stabilizing effect of the high pressure is not recognized by the previous authors.

THE PRESSURE DEVELOPMENT

It is interesting to notice how the pressure gradient develops throughout the region over which the insta-

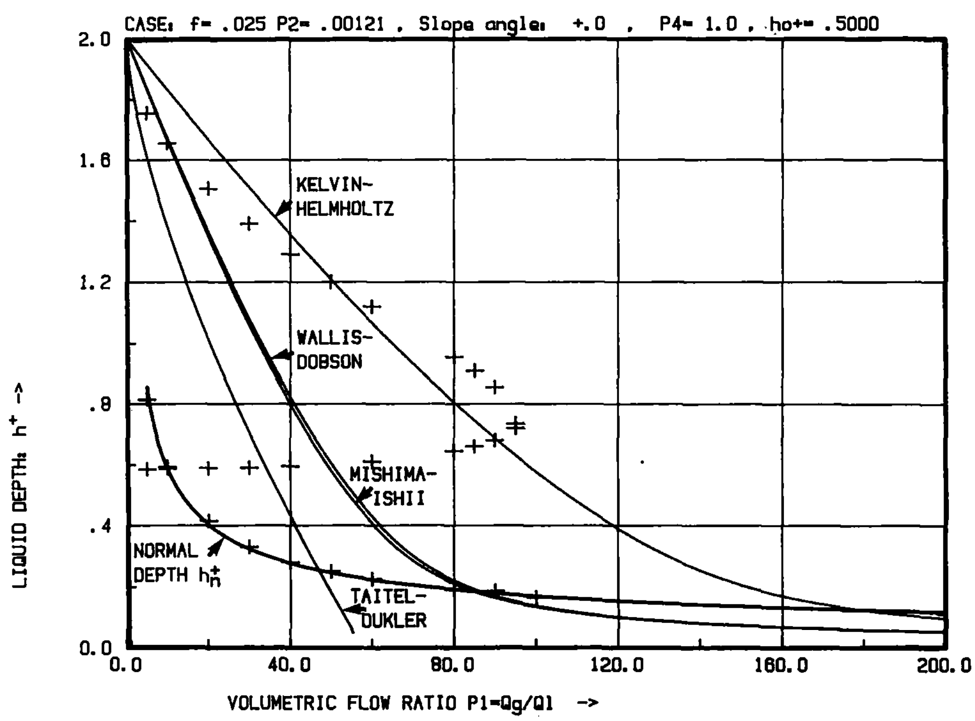

FIG. 12.

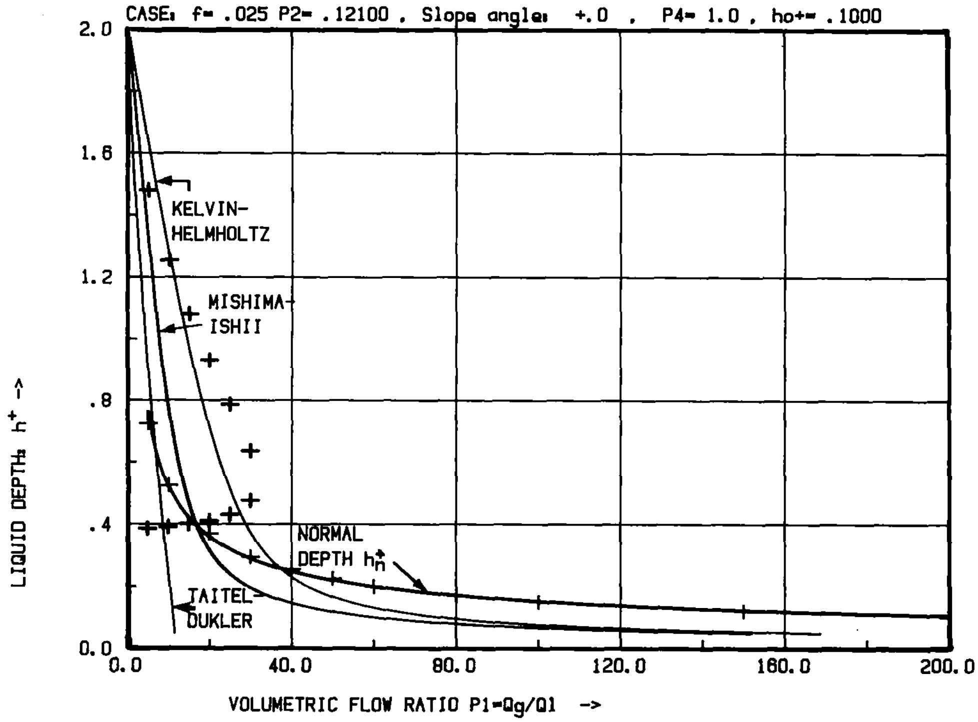

FIG. 13.

bility develops. This can be found from the basic equation in ref. [6] which is reproduced here as

$$\frac{d}{dx}\left(\frac{v_G^2}{2g}\right)+\frac{1}{\gamma_G}\frac{dp_G}{dx}+\frac{d}{dx}(h_m^{(G)})=0. \qquad (9)$$

It is a question of rather straightforward algebra to rearrange this equation and present it in the dimensionless form:

$$\frac{1}{\gamma_G}\frac{dp_G}{dx} = -h_0^+\left(\frac{P_1^2\sin\phi}{(F_G^+)^3}\frac{dh^+}{dx^+} + \frac{f}{8}\left(\frac{P_1^2}{(F_G^+)^2 R_G^+}+P_4\frac{(1/F_L^+ - P_1/F_G^+)^2}{s_i^+}\right)\right). \qquad (10)$$

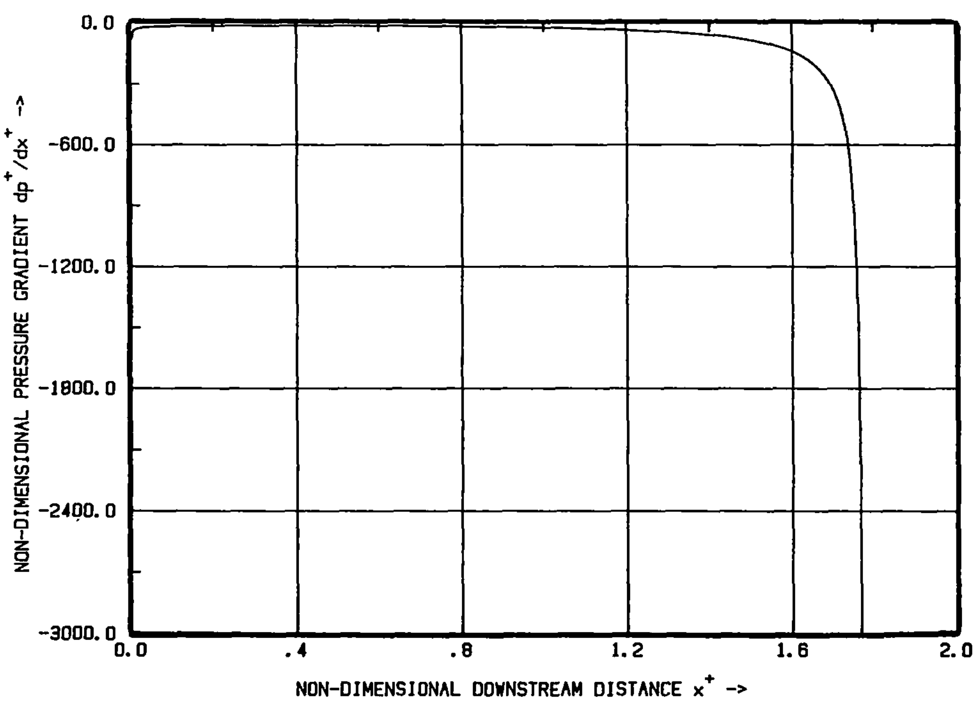

FIG. 14.

If one fixes attention on the situation illustrated in Fig. 9, one may illustrate the downstream development as the interface develops. This is done in Fig. 14, where it is obvious that the pressure gradient exhibits singular behaviour. This is in complete accordance with the results reported by Persen [9], where pressure recordings by passage of a slug are shown.

FINAL REMARKS

The preceding investigation of the stability of stratified flow has demonstrated that the stability question is so complex that one can hardly hope to incorporate all implications in one single instability criterion. Furthermore, it has been demonstrated that several physical processes, such as the force transmission (here expressed through P_4) and entrainment of liquid particles in the gas phase (here expressed through P_2), will have to be better known if the predictions are to conform with reality. The density ratio P_2 is defined through the actual (effective) densities of the two phases, whereby even engulfment of gas bubbles in the liquid phase will have to be considered.

The guesswork involved in the determination of the proper friction factor f to be used in the design of pipeline systems for single-phase flow cannot be avoided here either, and this may influence the stability criterion.

In spite of the uncertainties involved it is suggested that the analytic approach used here will also prove useful under other circumstances. It can easily be modified to incorporate porous walls (horizontal wells: gas/liquid) and the influence of long pipelines where friction will cause pressure drops of such magnitude that its influence on the stability can no longer be neglected. This investigation will be reported elsewhere.

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APPENDIX

The definitions of the different quantities appearing in the analysis are given as follows with reference to Fig. A1:

Liquid area	$F_L^+ = \phi - \sin(\phi)\cos(\phi)$
Gas area	$F_G^+ = \pi - F_L^+$
Wetted periphery for liquid	$P_L^+ = 2\phi$
Wetted periphery for gas	$P_G^+ = 2(\pi - \phi)$
Hydraulic radius for liquid	$R_L^+ = F_L^+/P_L^+$
Hydraulic radius for gas	$R_G^+ = F_G^+/P_G^+$
Interface length	$s_i^+ = 2\sin(\phi)$
The h^+, ϕ-relation	$h^+ = 1 - \cos(\phi)$.

All relations are dimensionless, lengths being divided by R, areas by R^2.

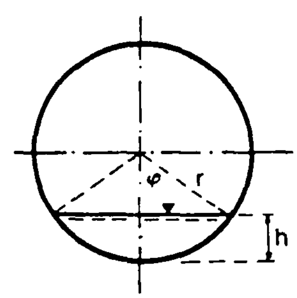

FIG. A1.